Properties Of Wave Motion In A Cylindrical Shell Is In Contact With A Viscous Fluid

Author's Details:
(1)**Safarov Ismail Ibrahimovich** Doctor of physical and mathematical sciences, professor, Bukhara Technological-Institute of Engineering, Republic of Uzbekistan.
(2)**Teshaev Mukhsin Khudoyberdiyevich-** Candidate of Physical Mathematical Sciences, docent, Bukhara Technological Institute of Engineering, Republic of Uzbekistan. 15, K. Murtazoyev Street.
(3)**Boltayev Zafar Ixtiyarovich –** researcher, Bukhara Technological Institute of Engineering, Republic of Uzbekistan.15 K. Murtazoyev Street (4)**Akhmedov Maqsud Sharipovich –** Senior researcher, Bukhara Technological- Institute of Engineering Republic of Uzbekistan. 15, K. Murtazoyev Street.

Abstract. *The distribution of natural waves in a cylindrical shell (elastic or viscoelastic) in contact with the viscous fluid is discussed. The problem to the solution of spectral problems with complex input parameters is reduced. Systems of ordinary differential equations by the Godunov orthogonal sweep method with a combination of Muller method is solved numerically. Dissipative processes in the mechanical system, an mechanical effect, describes the intensity of the flow of mechanical energy studied.*
Keywords*: shell, viscous fluid, spectral problem, frequency, phase velocity.*

Introduction.

The problem of wave propagation in a cylindrical shell filled or submerged liquid has important practical significance. The phenomenon of wave-like motion of the fluid in the elastic cylindrical shells attracted the attention of researchers [1,2,3,4,5,6,7]. In these works devoted to wave processes in a system of elastic cylindrical shell - ideal fluid, used and refined classical equations of shells, consider the influence of the radial and longitudinal inertial forces, considered the average density of the liquid or gas flow. In [8,9,10,11,12,13,14,15] analyzes the laws of wave process in an elastic shell with viscous fluid in the model of the linearized equations of hydrodynamics of viscous compressible fluid. Unlike other cylindrical shell system here (elastic or viscoelastic) and liquid (ideal or viscous) is considered as a dissipative heterogeneous mechanical system [16,18].

Statement of the problem.

We consider the natural oscillations of the infinite in length deformable (viscoelastic) cylindrical shell of radius R_1 with constant thickness h_0, density shell ρ, E_0- instantaneous envelope viscoelastic modulus, Poisson's ratio ν_0, filled with a viscous fluid with density ρ_0 in an equilibrium state. Fluctuations in the shell when exposed to internal pressure $\vec{p}(-p_1,-p_2,p_3)$, described by the equations [14,15,18,21]:

$$L\vec{u}-\int_0^t LR_E(t-\tau)\vec{u}(\vec{r},\tau)d\tau=\frac{(1-\nu_0^2)}{E_0h_0}\vec{p}+\rho_0\frac{(1-\nu_0{}^2)}{E_0}\frac{\partial^2\vec{u}}{\partial t^2}. \quad (1)$$

Here E_0- instantaneous elastic moduli, $\vec{u}=\vec{u}(u_r,u_\theta,u_z)$- displacement vector points of the middle surface of the shell, and for shells Kirhgora - Love he has a dimension equal to three $(u_r=u;\quad u_\theta=v;\quad u_z=w)$, and for the type of vector dimension Tymoshenko shells $\vec{u}$ is five, except here the axial, circumferential and normal movements added more angles of rotation normal to the middle surface in the axial and circumferential directions [18]; L- differential operator, for subordinating the shells in the hypothesis have Kirchhoff -Lyava view

$$L=\begin{pmatrix} \frac{\partial^2}{\partial x^2}+\frac{1-\nu}{2R^2}\frac{\partial^2}{\partial\varphi^2} & \frac{1+\nu}{2R}\frac{\partial^2}{\partial x\partial\varphi} & \frac{\nu}{R}\frac{\partial}{\partial x} \\ \frac{1+\nu}{2R}\frac{\partial^2}{\partial x\partial\varphi} & \frac{1+\nu}{2}(1+4a)\frac{\partial^2}{\partial x^2}+(1+a)\frac{\partial^2}{\partial\varphi^2} & \frac{1}{R^2}\frac{\partial}{\partial\varphi}-a(2-\nu)\frac{\partial^3}{\partial x^2\partial\varphi}-\frac{a}{R^2}\frac{\partial^3}{\partial\varphi^3} \\ \frac{\nu}{R}\frac{\partial}{\partial x} & \frac{1}{R^2}\frac{\partial}{\partial\varphi}-a(2-\nu)\frac{\partial^3}{\partial x^2\partial\varphi}-\frac{a}{R^2}\frac{\partial^3}{\partial\varphi^3} & \frac{1}{R^2}+a\left(\frac{\partial^2}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2}\right)^2 \end{pmatrix},$$

$a=h^2/12R^2$, $\{u\ \ v\ \ w\}^T$ - vector movement with axial, radial and circumferential components, respectively ("+" sign in front p_n and the "-" sign in front of the inertial member said last component that is considered to be a positive movement towards the center of curvature); $R_E(t-\tau)$–relaxation kernel. The amplitudes of the oscillations considered to be small, which allows you to record the basic relations in the framework of the linear theory. The system of linearized equations of motion of a viscous barotropic fluid can be written as [15]

$$\frac{\partial\vec{\vartheta}}{\partial t}-\nu*\Delta\vec{\vartheta}+\frac{1}{\rho_0^*}grad\,P-\frac{\nu*}{3}grad\,div\vec{\vartheta}=0$$

$$\frac{1}{\rho_0^*}\frac{\partial\rho*}{\partial t}+div\vec{\vartheta}=0;\quad \frac{\partial P}{\partial\rho^*}=a_0^2,a_0=const.$$

$$\dot{u}_z=\vartheta_z,\dot{u}_r=\vartheta_r,\ \dot{u}_\theta=\vartheta_\theta,$$

$$q_z=-p_{rz},q_r=-p_r,\ q_\theta=-p_{r\theta}.$$

$$p_{rz}=\mu^*\left(\frac{\partial\vartheta_z}{\partial r}+\frac{\partial\vartheta_r}{\partial z}\right);$$

$$p_{rr}=-p+\lambda^*\left(\frac{\partial\vartheta_r}{\partial r}+\frac{\partial\vartheta_z}{\partial z}+\frac{\vartheta_r}{r}\right)+2\mu^*\frac{\partial\vartheta_r}{\partial r}\ ; \qquad (2)$$

$$p_{r\theta}=\mu^*\left(\frac{1}{r}\frac{\partial\vartheta_z}{\partial\theta}+\frac{\partial\vartheta_\theta}{\partial r}-\frac{\vartheta_\theta}{r}\right).$$

Here, in the equations (2) $\vec{\vartheta}=\vec{\vartheta}(\vartheta_r,\vartheta_\theta,\vartheta_z)$ - vector fluid particle velocity; ρ^* and P-perturbations in the density and pressure of fluid; ρ_0^* and a_0 – density and speed of sound in the fluid at rest; ν^*,μ^* - kinematic and dynamic viscosity coefficient; for the second viscosity coefficient λ^* accepted ratio $\lambda^*=-\frac{2}{3}\mu^*$; $p_{rz},p_{rr},p_{r\theta}$ -costavlyaet stress tensor in a fluid. Equations (1), respectively, kinematic and dynamic boundary conditions, which, because of the thin-walled shell, we will meet on the middle surface (r = R). Equations (1) and (2) is closed gidrovyazkouprugosti relations system for a cylindrical shell containing a viscous compressible fluid. So for shells obeying Kirchhoff-Love hypothesis. To be joint study fluctuations membranes and fluid, harmonic in the axial coordinate z and decay exponentially over time, or time-harmonic and damped with respect to z.

2. Methods of solution. We accept the integral terms in (1) small, then the function $\vec{u}(\vec{r},t)=\psi(\vec{r},t)e^{-i\omega_R t}$, where $\psi(\vec{r},t)$- slowly varying function of time, ω_R- real constant. Next, using the freezing procedure [19], then the integral-differential equation (1) takes the following form

$$L\left[1-\Gamma^C(\omega_R)-i\Gamma^S(\omega_R)\right]\vec{u}=\frac{(1-\nu^2)}{E_0 h}\vec{p}+\rho_0\frac{(1-\nu_0{}^2)}{E_0}\frac{\partial^2\vec{u}}{\partial t^2}, \quad (3)$$

where for shells Kirchhoff - Love

$$L=\begin{pmatrix} \frac{\partial^2}{\partial x^2}+\frac{1-\nu}{2R^2}\frac{\partial^2}{\partial\varphi^2} & \frac{1+\nu}{2R}\frac{\partial^2}{\partial x\partial\varphi} & \frac{\nu}{R}\frac{\partial}{\partial x} \\ \frac{1+\nu}{2R}\frac{\partial^2}{\partial x\partial\varphi} & \frac{1+\nu}{2}(1+4a)\frac{\partial^2}{\partial x^2}+(1+a)\frac{\partial^2}{\partial\varphi^2} & \frac{1}{R^2}\frac{\partial}{\partial\varphi}-a(2-\nu)\frac{\partial^3}{\partial x^2\partial\varphi}-\frac{a}{R^2}\frac{\partial^3}{\partial\varphi^3} \\ \frac{\nu}{R}\frac{\partial}{\partial x} & \frac{1}{R^2}\frac{\partial}{\partial\varphi}-a(2-\nu)\frac{\partial^3}{\partial x^2\partial\varphi}-\frac{a}{R^2}\frac{\partial^3}{\partial\varphi^3} & \frac{1}{R^2}+a\left(\frac{\partial^2}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2}\right)^2 \end{pmatrix},$$

$$\Gamma^C(\omega_R)=\int_0^\infty R(\tau)\cos\omega_R\tau\,d\tau, \quad \Gamma^S(\omega_R)=\int_0^\infty R(\tau)\sin\omega_R\tau\,d\tau$$

- respectively, the cosine and sine Fourier transform of the relaxation of the core material. As an example, the viscoelastic material take three parametric relaxation nucleus

$$R(t)=Ae^{-\beta t}/t^{1-\alpha},$$

ρ – the density of the shell material; E – Young's modulus; ν – Poisson's ratio, $a=h^2/12R^2$. Let's move on to the dimensionless axial coordinate $\xi=x/R$ and multiply on R^2 system (3). The matrix of the resulting system has the form

$$L=\begin{pmatrix} \frac{\partial^2}{\partial\xi^2}+\frac{1-\nu}{2}\frac{\partial^2}{\partial\varphi^2} & \frac{1+\nu}{2}\frac{\partial^2}{\partial\xi\partial\varphi} & \nu\frac{\partial}{\partial\xi} \\ \frac{1+\nu}{2}\frac{\partial^2}{\partial\xi\partial\varphi} & \frac{1-\nu}{2}(1+4a)\frac{\partial^2}{\partial\xi^2}+(1+a)\frac{\partial^2}{\partial\varphi^2} & \frac{\partial}{\partial\varphi}-a(2-\nu)\frac{\partial^3}{\partial\xi^2\partial\varphi}-a\frac{\partial^3}{\partial\varphi^3} \\ \nu\frac{\partial}{\partial\xi} & \frac{\partial}{\partial\varphi}-a(2-\nu)\frac{\partial^3}{\partial\xi^2\partial\varphi}-\frac{a}{R^2}\frac{\partial^3}{\partial\varphi^3} & \frac{1}{R^2}+a\left(\frac{\partial^2}{\partial\xi^2}+\frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2}\right)^2 \end{pmatrix}. \quad (4)$$

Expanding equation (2) and (3) to coordinate FORMS, easy to see that the relations (2) - (3) break up into independent boundary value problems:

- torsional vibrations:

$$\frac{\partial p_{r\theta}}{\partial r}+\frac{2p_{r\theta}}{r}+\frac{\partial p_{\theta z}}{\partial z}=\rho_0^*\ddot{\vartheta}_\theta,$$

$$p_{r\theta}=\eta\left(\frac{\partial\vartheta_\theta}{\partial r}-\frac{\vartheta_\theta}{r}\right), \quad p_{\theta z}=\eta\frac{\partial\vartheta_\theta}{\partial r}, \quad (5)$$

$$r=R:\ Gh\frac{\partial^2 u_\theta}{\partial z^2}-(\rho_0 h\ddot{u}_\theta\pm p_{\theta r})=0, \quad G=\frac{E_0\left[1-\Gamma^c(\omega_R)-i\Gamma^s(\omega_R)\right]}{2(1+\nu_0)}.$$

$$r=0:\quad p_{r\theta}=0.$$

- longitudinally transverse vibrations:

$$\frac{\partial p_{rr}}{\partial r}+\frac{p_{rr}-p_{\theta\theta}}{r}+\frac{\partial p_{rz}}{\partial z}=\rho_0^*\ddot{\vartheta}_r$$

$$\frac{\partial p_{rz}}{\partial r}+\frac{p_{rz}}{r}+\frac{\partial p_{zz}}{\partial z}=\rho_0^*\ddot{\vartheta}_z$$

$$p_{rr} = -p + k_\eta div\,\dot{\vec{\vartheta}} + 2\eta\frac{\partial \vartheta_r}{\partial r},\ p_{\theta\theta} = -p + k_\eta div\,\dot{\vec{\vartheta}} + 2\eta\frac{\vartheta_r}{r},$$
$$p_{zz} = -p + k_\eta div\,\dot{\vec{\vartheta}} + 2\eta\frac{\partial \vartheta_z}{\partial z},\ p_{xs} = \eta\left(\frac{\partial U_z}{\partial z} + \frac{\partial U_s}{\partial r}\right), \quad (6)$$

$$\dot{\rho} + \rho_0 div\,\dot{\vec{\vartheta}} = 0, \qquad div\,\vec{\vartheta} = \frac{\partial \vartheta_r}{\partial r} + \frac{\vartheta_r}{r} + \frac{\partial \vartheta_z}{\partial z}, \qquad \frac{\partial p}{\partial \rho} = C_0^2$$

$$r = R: \qquad \frac{\partial^4 u_r}{\partial z^4} + \frac{C}{R}\left(\frac{u_r}{R} + \nu_0\frac{\partial u_z}{\partial z}\right) + p_{rr} + \rho_0 h\ddot{u}_r = 0,$$

$$C\left(\frac{\partial^2 u_r}{\partial z^2} + \frac{\nu_0}{R}\frac{\partial u_r}{\partial R}\right) - (p_{rz} \pm \rho_0 h\ddot{u}_z) = 0, \quad C = \frac{E_0 h}{1-\nu_0^2}\left[1 - \Gamma^c(\omega_R) - i\Gamma^s(\omega_R)\right]$$ Пусть wave

$$r = 0 \qquad p_{rz} = 0 \qquad u_r = 0.$$

process is periodic in z and fades over time, then is given a real wave number k, and the complex frequency is the desired characteristic value. Solution of (2) - (6) for the main unknowns satisfying cash above the limits for a relationship to time and coordinate z, should be sought in the form [17]

$$p_{rr} = \sum_{m=1}^{\infty}\sigma_{rm}\cos(m\varphi)e^{ikz-i\omega t},$$
$$p_{rz} = \sum_{m=1}^{\infty}\tau_{zm}\cos(m\varphi)e^{ikz-i\omega t},$$
$$p_{r\theta} = \sum_{m=1}^{\infty}\tau_{\varphi m}\sin(m\varphi)e^{ikz-i\omega t}, \quad (7)$$
$$\vec{u} = \sum_{m=1}^{\infty}\vec{u}_m(U_m\cos(m\varphi), V_m\sin(m\varphi), W_m\cos(m\varphi))e^{ikz-i\omega t},$$
$$\vec{\vartheta} = \sum_{m=1}^{\infty}\vec{\vartheta}_m(\vartheta_{rm}\cos(m\varphi), \vartheta_{\theta m}\sin(m\varphi), \vartheta_{zm}\cos(m\varphi))e^{ikz-i\omega t}$$

where $\sigma_r, \tau_z, \tau_\varphi$, U_m, V_m, W_m, $\vartheta_r, \vartheta_\theta, \vartheta_z$ - amplitude integrated vector - function; k is the wave number; C - the phase velocity; and ω – complex frequency; m - the circumferential wave number (wave number of the district), taking values m = 1, 2, 3 ... The case m = 0 axial symmetrical vibrations. This approach will seek a solution for every fixed value of the wave number of the district m independently.

In this way c, k, ω are known and the actual spectrum of the complex parameter to the type of task.

To clarify their physical meaning, consider two cases:

1) $\kappa = \kappa_R$; C = C_R +iC_i, then the solution (5) has the form of a sine wave in x, whose amplitude decays over time;

2) $\kappa = \kappa_R + i\kappa_I$; C = C_R, then at each point x fluctuations established, but x decay. In the case of axial symmetrical to the axis r = 0 conditions must be satisfied $p_{r\theta} = p_{rz} = 0$, ϑ_r =0.

In axisymmetric case on the axis r = 0 conditions must be satisfied $p_{r\theta} = p_{rz} = 0$, ϑ_r =0. If the outer surface r = R is assumed stationary, then $u_r = u_z = u_\varphi = 0$. The superposition of the solutions (7) forms an exponentially decaying time standing wave that describes the natural oscillations of the liquid and the final length of the cylindrical shell boundary conditions. With infinite length of the shell along the lines specified type of motion (7) is called a private or free oscillations. In the case of stationary time and the fading coordinate the process, in contrast, is known actual frequency ω, as desired be a complex wave number k. In contrast to their own, these fluctuations will be called Install. Actual values of the ω In the first case, and k, second frequency have the physical meaning of the process in time and the coordinate, respectively. Imaginary part - the rate of decay of wave processes over time and Z, respectively [17]. The value of 1/Imk sometimes defined as the interval of damped wave propagation. In the extreme case, the elastic propagation range is endless. The degree of attenuation of the wave process in the time period is characterized by the logarithmic decrement

$$\delta_c = 2\pi|\operatorname{Im}\omega| / \operatorname{Re}\omega \qquad (8,a)$$

decrement is similar to the spatial

$$\delta_y = 2\pi|\operatorname{Im}k| / \operatorname{Re}k \,. \qquad (8,b)$$

You can also introduce the concept of phase velocity of its own and steady motions

$$c_c = \frac{\operatorname{Re}\omega}{R}, c_y = \frac{\omega}{\operatorname{Re}k}$$

Quantities C_c and C_y have a physical meaning zero state at their own speeds and steady oscillations, respectively, and, in contrast to the elastic (real) case, do not coincide with each other at the same frequencies. Two types of oscillation (own and install) can put two different formulations of the problem. And in the case of non-stationary, namely Cauchy problem for an infinite shell and boundary value problem for a semi-infinite interval changes Z. In both cases the solution is using the integral transformation of the solutions of the corresponding stationary problems. Thus, in the case of the Cauchy problem, the main vector of unknowns $\overline{Y}^c$ it can be in a superposition of waves

$$\overline{Y}^c = (r,z,t) = \sum_n \int_{-\infty}^{\infty} Y_n^c(r,k)\exp[t(kz - \varpi_n(k)t)]dk \quad , \qquad (9)$$

where the vectors $\overline{Y}_n^c$ are their own form of natural vibrations problem, normalized so that the spatial spectrum of the initial disturbance Fourier $\overline{f}(r,z) = \overline{Y}^c(r,z,0)$ forms a linear combination

$$\overline{f}(r,z) = \int_{-\infty}^{\infty} F(r,k)e^{ikz}\, dk, \qquad \overline{f}(r,k) = \sum_n \overline{Y}_n^c(r,k) \;. \qquad (10,a)$$

Similarly, the main vector of unknown $\overline{Y}^y$ boundary value problem is calculated according to the expression

$$\overline{Y^y}(r,z,t) = \sum_n \int_{-\varpi}^{\varpi} \overline{Y}_k^y(r,\omega)\exp[ik(\omega)z - \omega t]d\omega \quad (10,b)$$

where $\overline{Y}_k^y$ - shaped stationary vibrations, linear combination of which should form a Fourier spectrum given boundary perturbation

$$\overline{q}(r,t) = \overline{Y}^y(r,0,t), \overline{q}(r,t) = \int_{-\infty}^{\infty} q(r,\omega)e^{-i\omega t}d\omega, \;\; \overline{q}(r,\omega) = \sum_n \int_{-\infty}^{\infty} Y_n^y(r,\omega)$$

Obviously, the solutions (2.7) are meaningful only if there exist (10a) and (10, b). So there are four possible variants of stationary movements, which are discussed below: own and steady oscillations shell system - the fluid inside and outside the shell-liquid [15]. Substituting the solution (7) in the system of differential equations (2) - (6) we obtain a system of ordinary differential equations with complex coefficients, which is solved by Godunov orthogonal sweep method with a combination of Muller [17] in the complex arithmetic.

3. The vibrations of cylindrical shells with an ideal fluid.

To describe the vibrations of a shell with a perfect fluid, used in the general equations of the theory of thin shells in the movements and perfect zhitkost [15]:

$$\sum_{k=1}^{3}(L_{jk}u_k+\rho h\ddot{u}_j-\delta_{3j}p\big|_{r=R})=0,\ j=1,2,3$$
$$\Delta p-\frac{1}{c^2_\infty}\ddot{p}=0,\ (p,r)_{r=R}=-\rho_\infty u_3, \qquad (11)$$

where u_j - components of the vector the shell displacements, $L_{j.k}$ – differential operators of the theory of shells, Δ - Laplace operator, δ_{3j} (j=1,2,3)- Kronecker, p unsteady hydrodynamic pressure, ρ_∞, c_∞ - density and sound velocity in the liquid; ρ_0, R, h_0 material density, radius and thickness of the shell.

$$\begin{Bmatrix} u_k(R,z,\theta,t) \\ p(r,z,\theta,t)\end{Bmatrix}=\begin{Bmatrix} u_k(R) \\ p(r)\end{Bmatrix}\exp(i(kz+m\theta+\omega t)) \qquad (12)$$

In the case of shells with an ideal fluid, then the solution of equation (11) is expressed in terms of special Bessel functions. Substituting expressions (12) in (11) leads to the following relation between the pressure on the liquid surface and its the shell deflection [3,4,5]

$$p(R)=\omega^2\rho_\infty\frac{I_n(\beta R)}{\beta I_n'(\beta R)}U_3,\ \ \beta^2=\left(\frac{\pi}{L}\right)^2-k^2,\quad k=\frac{\omega}{c_\infty},$$

Here I_n - modified Bessel function of the 2nd kind.

Attitude $I_n(\beta R)/\beta I_n'(\beta R)$ It has sleduyuўie asymptotic expression ([3,4,5]):

a) for small values of the argument

$$\frac{I_n(\beta R)}{\beta I_n'(\beta R)}=\begin{cases}\frac{2R}{(\beta R)^2}\left(1+\frac{(\beta R)^2}{8}\right)+0\left[\left(\frac{\beta R}{2}\right)^4\right], & n=0,\\ \frac{R}{n}\left[1-\frac{(\beta R)^2}{2n(n+1)}\right]+0\left[\left(\frac{\beta R}{2n}\right)^4\right], & n>0,\end{cases} \qquad (13,a)$$

b) for large values of the argument

$$\frac{I_n(\beta R)}{\beta I_n'(\beta R)}=\frac{1}{\beta}(1+0\left[\left(\frac{1}{\beta R}\right)\right], \qquad (13,b)$$

the principal terms of the expansions (13a), (13 b) accept as an approximation expressions uchitivayuschih inetsionnoe impact of fluid in the equations kolibany the shell. In [5,6,7] similar expressions have been used as an approximate solution of various problems as a stationary and non-stationary dynamic interaction of membranes with a liquid medium. The solution of the original system of differential equations of vibrations of a shell taking into account the expression (11) - (13) is reduced to the determination of the roots of the characteristic equation

$$\begin{vmatrix} f_{11}-\rho_0\dfrac{(1-\nu_0^2)\omega^2}{E_0} & f_{12} & f_{13} \\ f_{12} & f_{22}-\rho_0\dfrac{(1-\nu_0^2)\omega^2}{E_0} & f_{23} \\ f_{13} & f_{23} & f_{33}-\rho_0\dfrac{(1-\nu_0^2)\omega^2}{E_0} \end{vmatrix}=0, \qquad (14)$$

where

$$f_{11}=\lambda^2+\frac{1-\nu_0}{2R^2}n; \qquad f_{12}=\frac{1-\nu_0}{2R^2}\lambda n; \quad f_{13}=\frac{-\nu_0}{R}\lambda; \quad \lambda=\pi/L,$$

$$f_{22}=\frac{1-\nu_0}{2}\lambda^2+\frac{n^2}{R^2}+\frac{h^2}{12R^2}\left[\frac{n^2}{R^2}+2(1-\nu_0)\lambda^2\right];$$

$$f_{23}=-\frac{h_0^2}{R^2}\left[1+\frac{h_0}{12}\left(\frac{h_0^2}{R^2}+2(1-\nu_0)\lambda^2\right]; f_{33}=\frac{1}{R^2}+\frac{h_0^2}{12}\left[\frac{h_0^2}{R^2}+\lambda^2\right]^2,$$

and the value μ, which characterizes the ratio of the mass of the liquid attached. In the case of the shell vibrations without contact with the fluid, this equation reduces to the known algebraic equation of the eighth degree in accounting When the liquid in the exact formulation (11) is obtained by implicit transcendental equation with respect; approximate representation (13) provides, as in the case of "dry" shell eighth degree polynomial (except n=0, the polynomial is the ninth degree). In all cases, the condition for existence of a solution (12) of the traveling wave type is the positivity of one of the real roots of the equation. For approximate representations of this condition can be rigorously proved. Write out the equation (13) allows for fluid analysis influence the characteristic parameter λ oscillation of a shell defining the wave formation along the generator the shell.

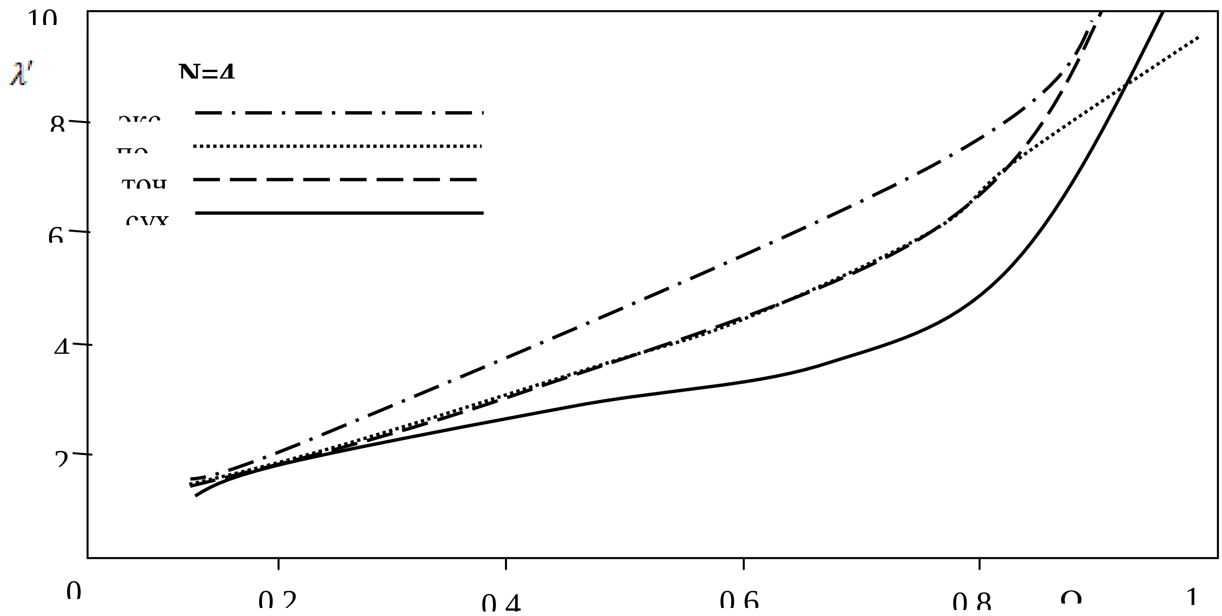


Figure 1.a. The dependence of the λ' on the frequency Ω for approximations (13).

Due to the fact that the range of analyzed frequencies is limited to the area of applicability of the original equations of the linear theory of shells, comparing the results of exact and approximate solutions also allows you to set the best ranges of applicability of each of these concepts. We will monitor the behavior of the smallest (in the range of low frequencies - the only) positive root of the characteristic equation (14), which correspond to the shape of the shell predominantly bending vibrations of a general nature, we estimate the effect of the main parameters of the problem.

We believe that within the shell is fluid, a condition which can range from a highly rarefied gas to water. We confine ourselves to the frequency range below the ring ($0<\Omega=\omega R\sqrt{\rho/E}<1$), in which the selection of the relative thickness of the shell has little effect on the value of λ (Naturally, we are talking about with thin shells h/ R<0,01).
The vast majority occur in practice shells has a characteristic parameter $\frac{h}{R}$ thickness lying in the range $\left[\frac{1}{100}\ \ \frac{1}{50}\right]$. Therefore, as starting problem parameters take data from the work [17,21]:

$$h/R=1/300,\ \nu=0{,}34,\ \rho_{\infty}/\rho=4{,}3.10^{-3},\ C_{\infty}\sqrt{E/\rho}=0{,}062.$$

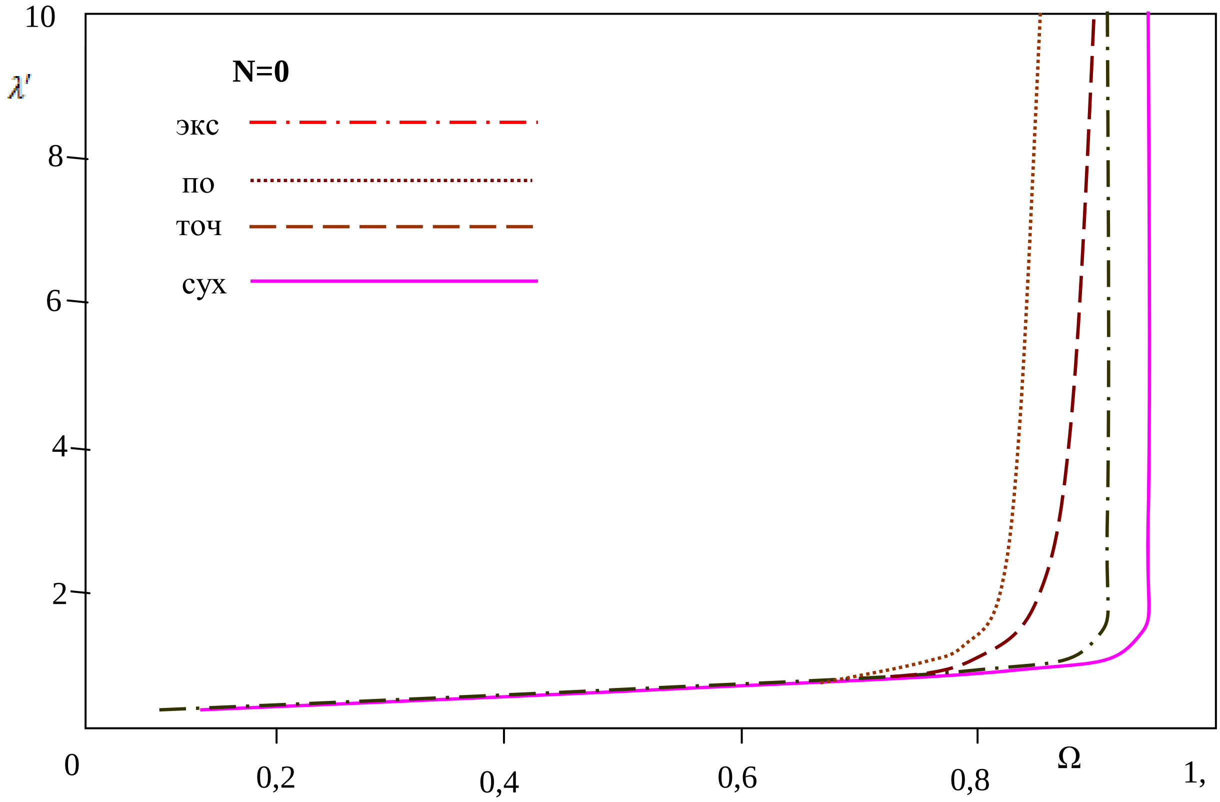


Figure 1.b. The dependence of the λ' on the frequency Ω for approximations (13).

Figure 1, a, shows the calculated dependence of the dimensionless parameter λ =λ/R от Ω dimensionless frequency in axially symmetric case and essentially forms the shell oscillations with n = 4. Designed and also shows the dispersion relations λ from Ω to an empty shell. From the elimination of these curves shows that the range of applicability of the approximation (13) corresponds to the lower frequencies, and it expands with the growth. For example, if n = 0,10% and a difference between the curves SPOT (accurate) and Paul (empty shell) has been observed with frequency Ω = 0,75, then the n =4 gives an approximation of the correct function of the fluid pressure on the surface of the shell has to frequency Ω = 0,9. The approximation (13), as one would expect, has a scope of applicability of the best at higher frequencies. It is important to note that the simultaneous use of representations (12) and (13) allows you to get a wide frequency range of the lower and upper bounds for the exact rate, "added mass of liquid".

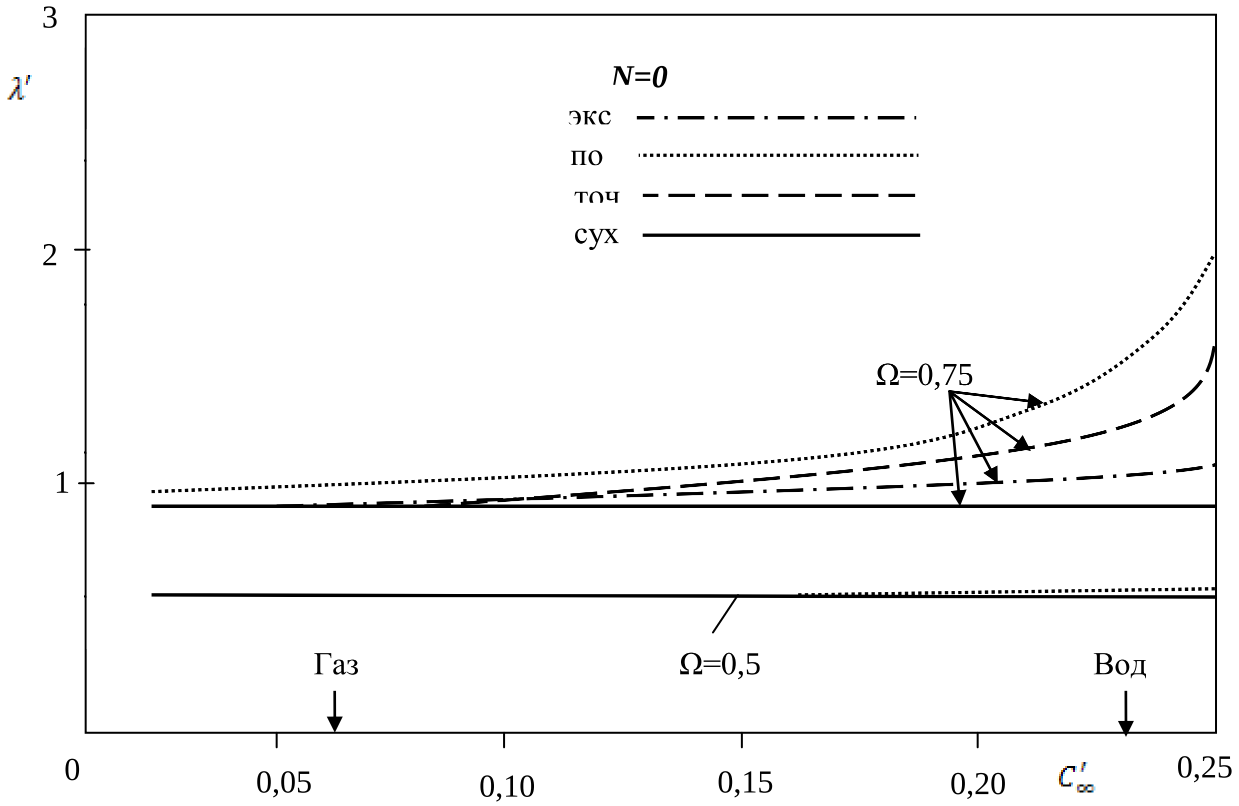


Figure 1, c. The dependence of the λ' a parameter $c_\infty/\sqrt{E/\rho}$ for n=6.

Thus, the above approach are complementary areas of its best applicability and can be used to determine the exact boundaries of the solution.

Fig. 1. a and,1. b illustrate parameters wobble $c_\infty/\sqrt{E/\rho}$ и ρ_∞/ρ on index at λ Ω typical values of frequency in the interval [0.5, 0.8] in the n =0,6. To find out which of these parameters has the greatest impact on the value of X, applied purely mathematical device: a change in one of them at a fixed value of another. From Figure 1, a. the effect of changes can be seen weak $c_\infty = c_\infty/\sqrt{E/\rho}$ on harakterichesky parameter λ, which allows in many cases to use a model of an incompressible fluid ($c_\infty = \infty$). At the same time, of Figure 1, c. that the value of ρ_∞/ρ Influencing factor in the determining. In conclusion, we note that both of the

submission are entitled to sushestvovanie in solving problems of joint shell with a liquid vibrations. The approximation (13) can give good results in the study of vibrations at low frequencies (or low characteristic parameters) in the solution, for example, stability problems or learning enveloped mode shapes.

4. Torsional vibrations

After performing in (5) the change of variables (7) permitting relations describing stationary torsional vibration system shell liquid, formulated in the form of spectral boundary value problem for a system of two ordinary differential equations

$$\frac{d\tau_\varphi}{dr} = -(\rho_0\omega^2 - i\eta^2\kappa^2\omega)v - \frac{2\tau_\varphi}{r}$$
$$\frac{dv}{dr} = \frac{v}{r} + \frac{i}{\omega\eta}\tau_\varphi \qquad (15)$$
$$r = R_1 : h(Gk^2 - k\rho_0\omega^2)v \pm \tau_\varphi = 0$$
$$r = 0 : \tau_\varphi = 0$$

First investigate fluid vibrations in the walls. Equations (15) can be converted to a single equation for the displacement *v*

$$\frac{d^2v}{dr^2} + \frac{dv}{rdr} + (-k^2 + i\frac{\omega}{v^*} - \frac{1}{r^2})v = 0; \quad v^* = \frac{\eta}{\rho_0^*} \qquad (16)$$

The solution of equation (16) bounded at r = 0 has the form

$$v = A_1 J_1(r\sqrt{-k^2 + i\frac{\omega}{v^*}}) = 0 \quad . \qquad (17)$$

Where J_1, Bessel function of the first order, and A is an arbitrary constant. Given the immobility of a shell, we obtain the dispersion equation

$$J_1(R_1\sqrt{-k^2 + i\frac{\omega}{v^*}}) = 0 \qquad (18)$$

from whence

$$\omega_n = -i(v^*k^2 + \Gamma_m^2) \qquad (2.16)(19)$$

in the case of natural oscillations and

$$k_n = \sqrt{-\Gamma^2 n + i\frac{\omega}{v^*}} \qquad (20)$$

in the case of steady-state oscillations. Here, through the Γ_n marked the roots of Bessel functions, referred to R. As can be seen from the formulas (18), (19) own movement always aperiodic time, with anchor points are fixed (the phase velocity C_o=0), while the steady movement are oscillatory in nature, and hotspots move with the speed $C_{y,}$ monotonically increasing from zero to infinity or to increase the viscosity v^* . These characteristics of viscous fluid motion will be shown in the subsequent more complex examples. Let us now consider the relation (15) for the case of an internal arrangement of the liquid. This problem can be solved in the same way using special features and have the dispersion equation

$$-k^2 + \frac{\omega^2}{a^2} + \frac{\omega v^*}{a^3\tilde{\rho}\tilde{h}R^2} + (z\frac{J_0(z)}{J_1(z)} - 2) = 0 \qquad (21)$$

which was first obtained by A. Guzya [8] Here we have introduced new designations

$$\tilde{p}=\frac{\rho^*}{\rho_0};\tilde{h}=\frac{h}{R_1};z=R_1\sqrt{-k^2+i\frac{\omega}{v^*}};\ a=\sqrt{\frac{G}{\rho_0}}$$

shell shear wave velocity: J_o-Bessel function of zero order.

The direct solution of equation (21) comes up against certain difficulties caused by the need to compute Bessel functions of complex argument. Therefore we examine (21) by means of asymptotic representations of these functions for small and large arguments z. The smallness of z occurs when low-frequency vibrations. According to the known expansions J_0 and J_1 power series

$$J_0=1-\frac{z^2}{4}=...;\ J_1(z)=\frac{z}{2}(1-\frac{z^2}{8}+...); \qquad (22)$$

Hold in the expansions (22) only the first term, we obtain

$$-k^2+\frac{\omega}{a^2}=0$$

dispersion equation of torsional vibrations or dry shell filled with an ideal fluid, keeping in (22) in the first two terms, we have the equation

$$-k^2+\frac{\omega^2}{a^2}+i\frac{\omega v^*}{4a^2\tilde{p}\tilde{h}}(k^2-i\frac{\omega}{v^*})=0 \qquad (23)$$

which is the root, for example, in the case of steady-state oscillations is given by

$$k=\frac{\omega}{a}\left[(1+\frac{1}{4\tilde{p}\tilde{h}})/(1-\frac{\omega v^*}{4a^2\tilde{p}\tilde{h}})\right]^{1/2}. \qquad (24)$$

The physical interpretation of the equation (21) given below. Consider now the situation where z is sufficiently large, which corresponds to a high-frequency vibration and low viscosity. In this case, the asymptotic formulas for the Bessel functions have the form

$$J_0(z)\cong(\frac{2}{\pi z})^{1/2}\cos(z-\frac{\pi}{4})$$

$$J_1(z)\cong(\frac{2}{\pi z})^{1/2}\sin(z-\frac{\pi}{4})$$

From (23) and (24) it is easy to prove that for a sufficiently large positive imaginary part z: $J_0(z)/J_1(z)\cong -i.$ Substituting (15) and further assuming smallness v^* compared with the value of $\frac{\omega}{k^2}$, to obtain an approximate dispersion equation, which is also contained in the [8]

$$-k^2+\frac{\omega^2}{a^3}(1+\sqrt{\frac{v^*}{\omega}}\frac{\tilde{p}}{\tilde{h}R}\frac{l+i}{1.41})=0 \qquad (25)$$

Whence, at aspiration the viscosity coefficient v^* to zero (and also at aspiration ω to infinity), we have a trivial result $\frac{\omega}{k}\to 0$, which was obtained at low ω from equation (23).

Equation (25) when an unacceptably high viscosities. In this case, the phase velocity with unlimited increases with ω. This example shows inconsistencies different asymptotic estimates in the mid-frequency vibrations. Thus, when analyzing wave processes asymptotic methods in the first approximation can not establish the limits of applicability of the formulas obtained, as well as to evaluate the error of calculations. In this paper for solving spectral problems using a direct numerical integration of resolving the type of (15) using the method of orthogonal shooting in complex arithmetic. This approach avoids the aforementioned difficulties associated with the calculation of Bessel functions of complex argument. Another advantage is due to the specificity of the orthogonal sweep method, which is due to the procedure orthonormality can solve highly rigid system with a boundary layer. As a result of a numerical study has found that the problem of natural oscillations (15) allows no more than one complex value w, corresponding to vibrations of a shell together with the adjacent layers of fluid. The rest found their own values were purely imaginary. They correspond to the aperiodic movements of the liquid at almost fixed shell.

Ownership corresponding to complex values, are also complex, that is, the phase of joint fluid and membrane vibrations are not the same along the radius. In the case of steady-state oscillations all the calculated eigenvalues k and their own forms were complex.

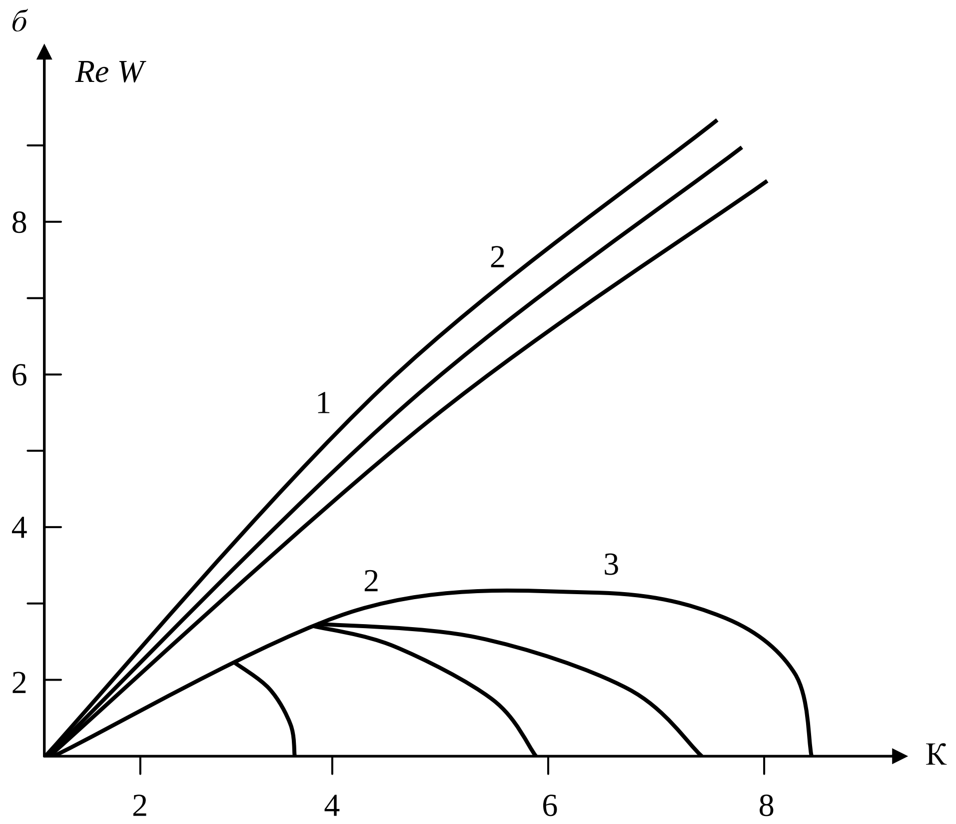


Fig.2 a. Addiction Reω the wave number k

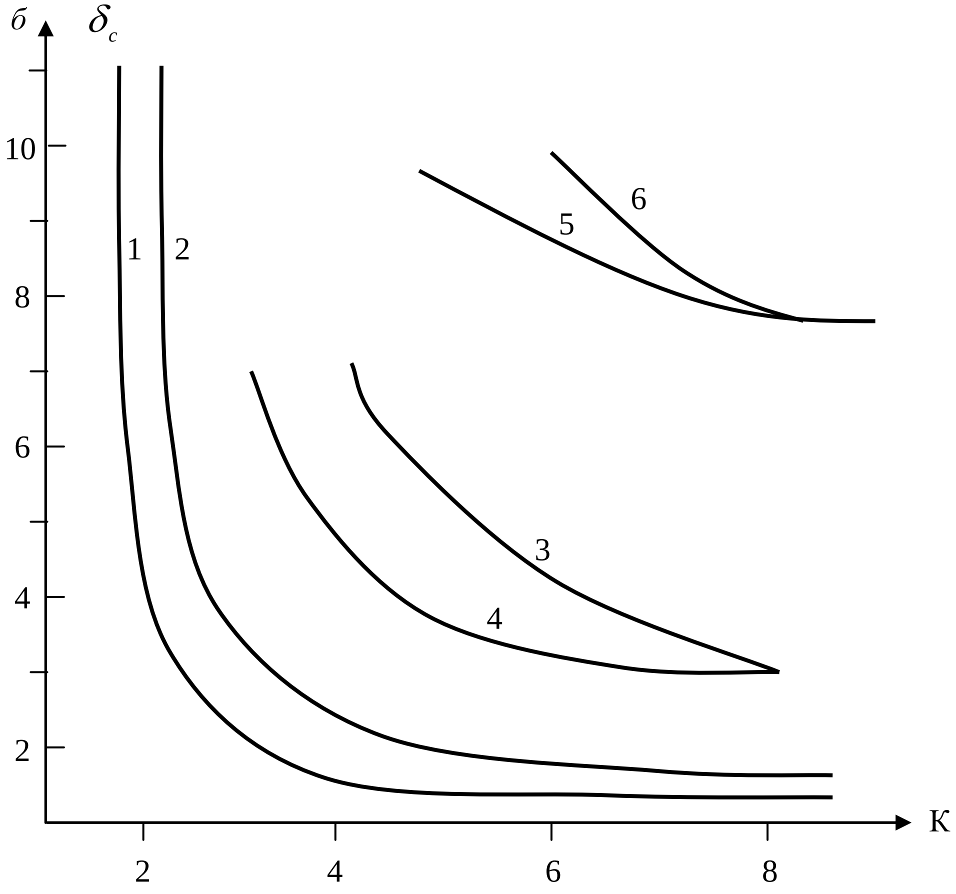


Fig.2 b. Addiction δ_c the wave number k

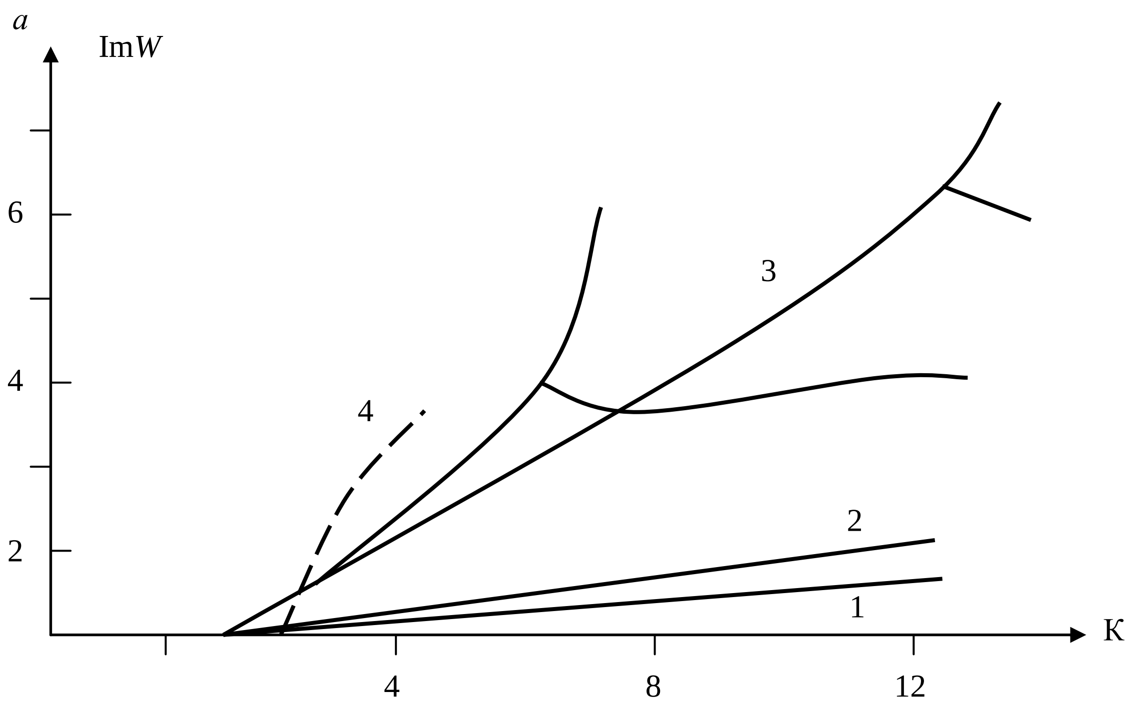


Fig.3 a. Addiction Imω the wave number k

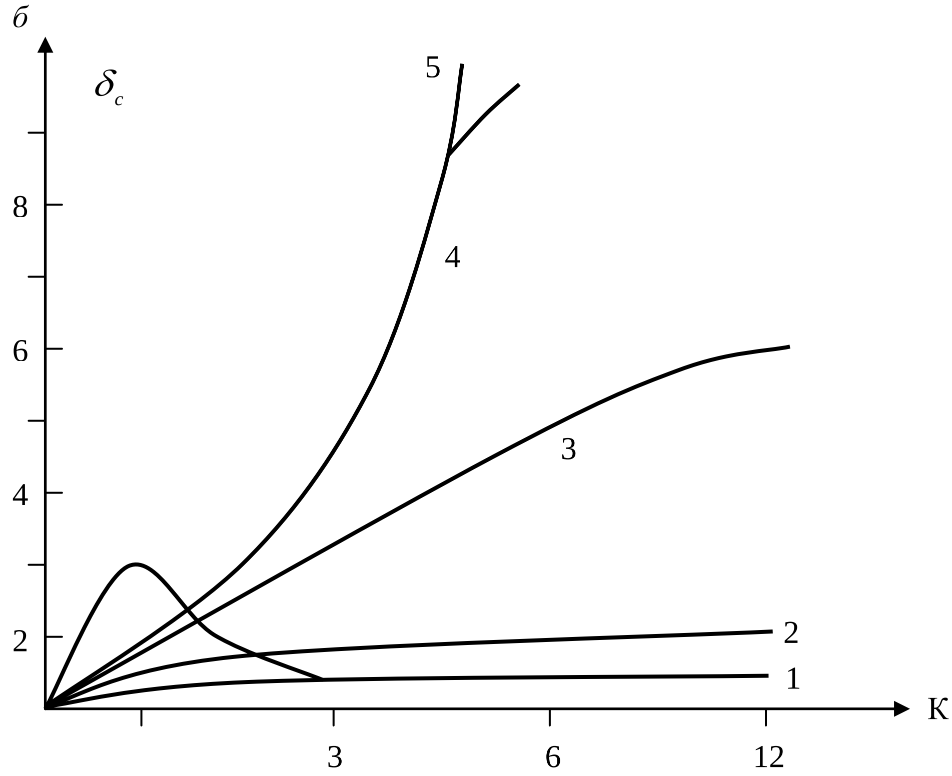


Fig.3. b. Addiction δ the wave number k

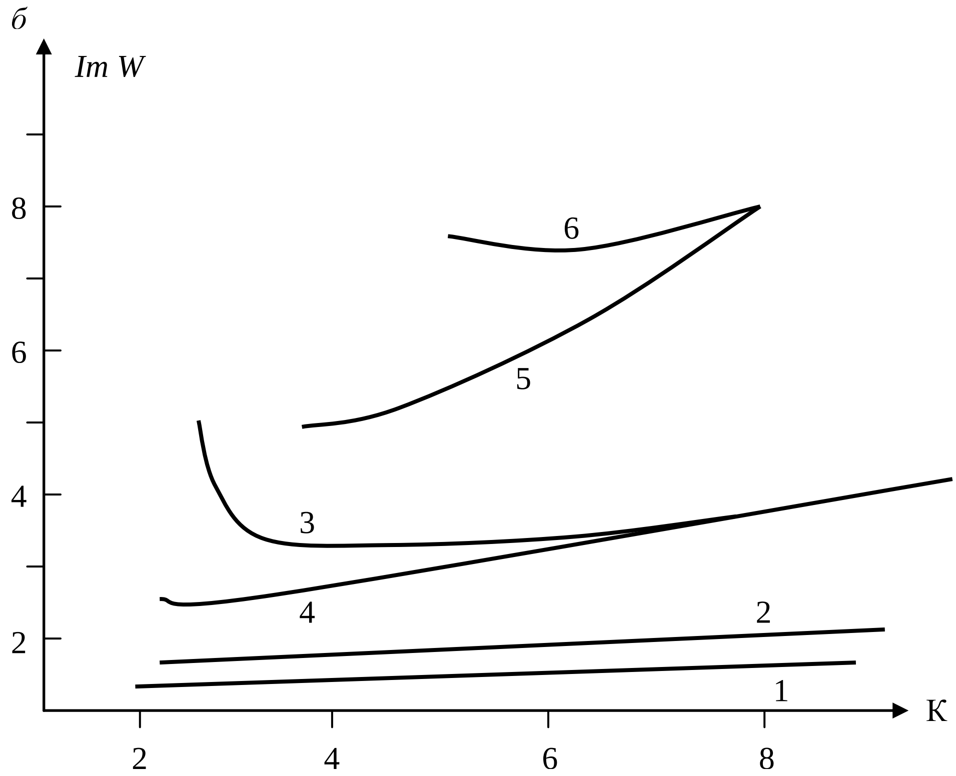
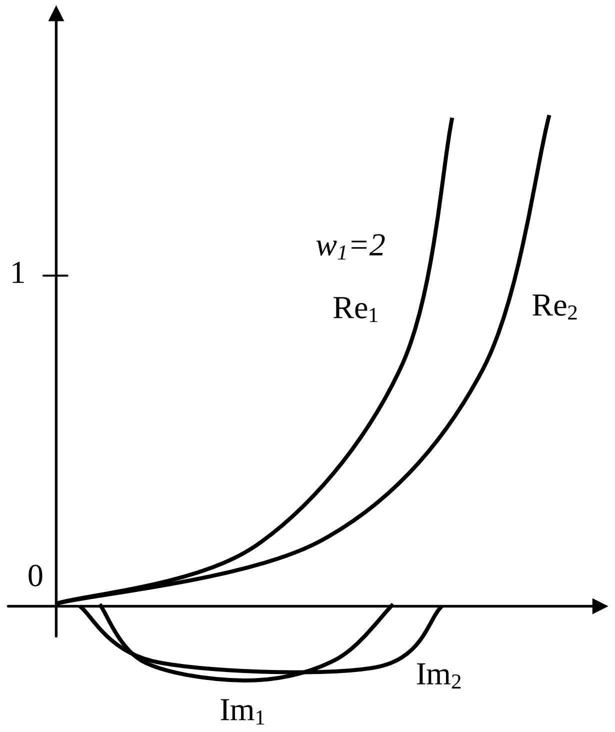
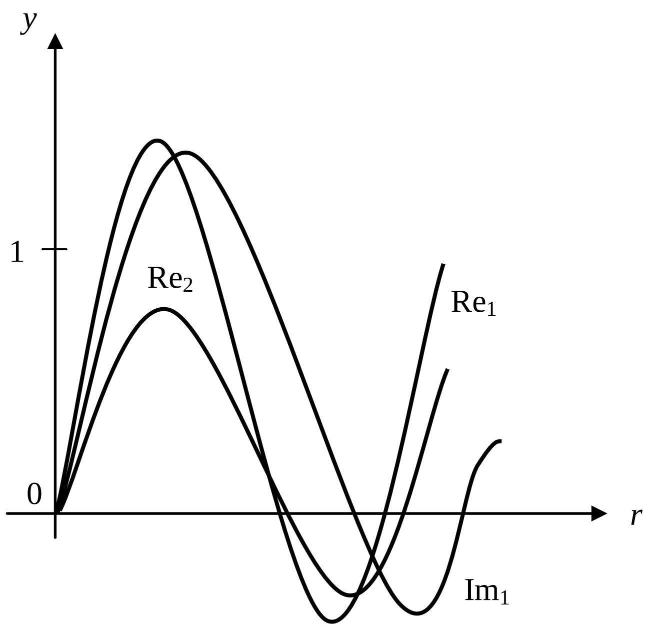


Fig.4. Addiction Im ω the wave number k



a)



б)

Fig.5. Addiction ν the wave number r

a. ω=2, η=0,018;***б)*** ω=2, η=0,0018.

Numerical results. Consider the case of natural oscillations when the shell is filled with liquid. Figure 2a, b, respectively, Figure 3 shows dispersion curves depending Re ω, Im ω, σ the wave number k-first mode, in which the damping coefficients of the lowest, and the eigenvalues are complex Bat.

In accordance with the numbering of the graphs were asked four different coefficient values η 1) 0.0009: 2) 0.0018 3)0.15 4)0.018 while other parameters. Figure 2b shows the own forms *Rev* values for k equal to 1 and 8 respectively. It is easy to notice the characteristic difference in the behavior of the dispersion curves 1,2 and 3,4. In the last two cases, there is a value of the wave number beginning with a certain magnitude to accept only purely imaginary values corresponding to aperiodic motion of the system. For curves 1.2 with a lower coefficient of viscosity real part of the eigenvalues $\mathrm{Re}\,\omega$ is different from zero in all wave numbers and the damping rate has a finite limit at infinity. The greater the viscosity, the earlier start aperiodic motion (curves 3,4) and the higher limit of the damping decrement (curves 1,2). It follows that there is a minimum critical ratio of viscosity ηk, above which a zone of high wave numbers of the first mode, there are aperiodic wave number. it was found that the critical values of the viscosity coefficient The numerical experiment η_k, it ranges.

Analyzing the dependence of the energy dissipation of the wave number, it should be noted that there are two opposing trends. With the increasing wave of a fixed amplitude ϑ According to (6) of linearly increasing shear stresses $p_{z\varphi}$: c another, as evidenced by Fig. 2.b, simultaneous localization of fluid motion amplitudes near the shell, resulting in a decrease in mass of the liquid involved in the movement, as well as shear stresses $p_{r\varphi}$.

The difference in behavior of curves 1.2 and 3.4 due to the fact which of the two trends prevail. At small wave numbers are observed linear dependence of its own form v on the radius, that is involved in the movement of the entire mass of the liquid. As the central portion of k growth liquid begins to "not have time" for the shell variations which leads to localization of the amplitudes. localization speed depends on the viscosity of the fluid. If localization is slow, then starting with some k (as a result of stress increase $p_{z\varphi}$) proper motions are aperiodic (curves 3,4). If the average in terms of the amplitude of the oscillations of the liquid decreases quickly, the movement will always keep the oscillatory character (curves 1,2). This large wave of voltage $p_{r\varphi}$ prevail over voltages $p_{z\varphi}$, and increases as the location. By virtue of the latter that the damping coefficient always increases with increasing k. The linear dependence of the form ϑ the radius for small k also shows the performance of the hypothesis of flat sections, which is based on the elementary theory of viscoelastic bars. With the help of the Ritz method parameters can be found Voigt rod model and define the limits of applicability of this model in the framework of the hydrodynamic theory, but for a narrow class of direct rods of circular cross section. The variational equation of the principle of virtual displacements, the equivalent ratio it looks.

$$\int_v h\left(\frac{\partial u_\varphi}{\partial z}\delta\frac{\partial u_\varphi}{\partial z}+\rho_1\frac{\partial u_\varphi}{\partial z}\delta u_\varphi\right)R_1 d\varphi dz-\int_v\left(\sigma_{r\varphi}\delta\varepsilon_{r\varphi}+\sigma_{z\varphi}\delta\varepsilon_{z\varphi}+\rho_0\frac{\partial^2 u_\varphi}{\partial z^2}\delta u_\varphi\right)rd\varphi drdz=0 \quad (26)$$

Choosing a linear function u_φ as base

$$u_\varphi(r,z,t)=\varphi(z,t)r, \qquad (27)$$

substituting (27) into (26) and receive standard procedure where the parameters β and a_0 expressed in terms of the polar moments of inertia of the shell I_1 and liquid I_0 in the following way

$$\beta = \frac{\eta I_0}{G I_1}; a_0 = a/(1+\frac{I_0}{\tilde{\rho} I_1})^{\frac{1}{2}}$$

Equation (26) describes the torsional vibrations in a viscoelastic rod Voigt according to relations

$$\left(1+\beta\frac{\partial}{\partial t}\right)\frac{\partial^2\varphi}{\partial z^2} = \frac{1}{a_0^2}\frac{\partial^2\varphi}{\partial t^2}, (28)$$

where β - viscosity.

The solution (28) is represented as

$$\varphi(z,t) = \varphi_0 \exp(i(kz-\omega t)) \ .$$

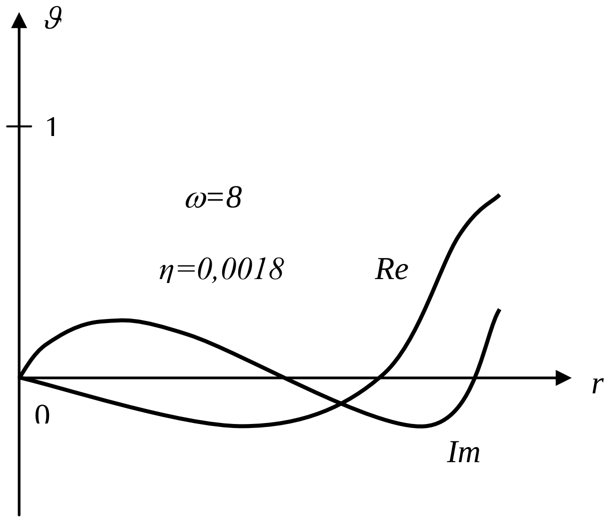
Where φ_0 satisfies the following relations

$$a_0^2\ \kappa^2(1-i\omega\beta)-\omega^2 = 0. \qquad (29)$$

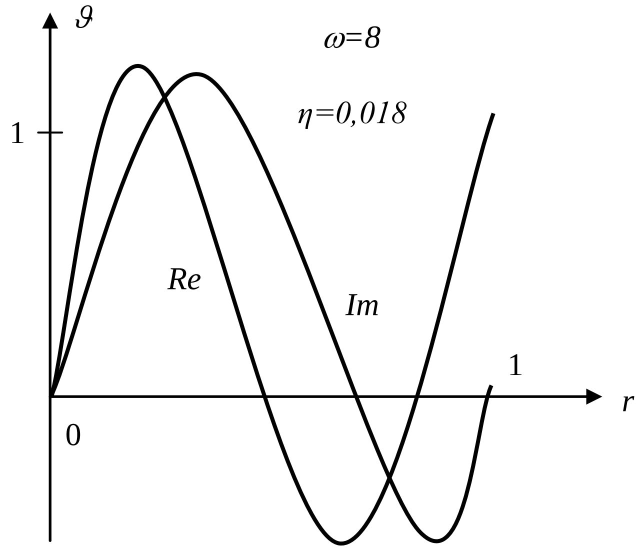
Taking into account the ratio I_1/I_0=4h, is easy to see that (29) coincides with the equation (25), which was obtained in the asymptotic solution of the problem (19) for the low-frequency vibrations. Fig. 2a, Figure 3a and dashed lines shows dispersion curves of natural vibration results from equation (29). As the drawings, a satisfactory agreement dotted and solid lines are observed in the region of small wave numbers, the upper limit of which is greater than in the case of a unit and increases with fluid viscosity. The shortwave discrepancy occurs due to the localization of the amplitudes of the oscillations near the shell. Small wave numbers correspond to the natural oscillations of the long end pipes. Let us analyze the steady oscillation of a shell filled with liquid. Figure 4, 5.6. are the dispersion curves and mode shapes for the two viscosity values (below and above critical) 1) 0.0018, 2) 0.018 and the same values of the other parameters, as in (26). In the first case, a relatively low viscosity, the calculation results are in good agreement with the asymptotic solutions of Guzya (21) at high frequencies.

Table 1

ω	1	2	3
5	8.956+11.200	17.125+12.259	8.955+11.135
10	17.125+12.192	9.527+11.244	18.085+11.759



a)



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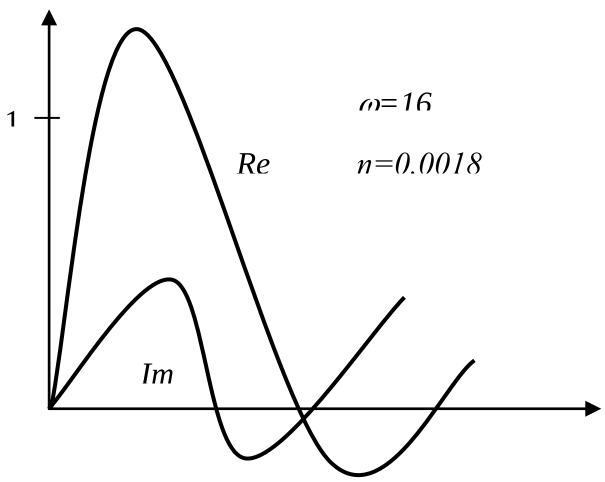

c)

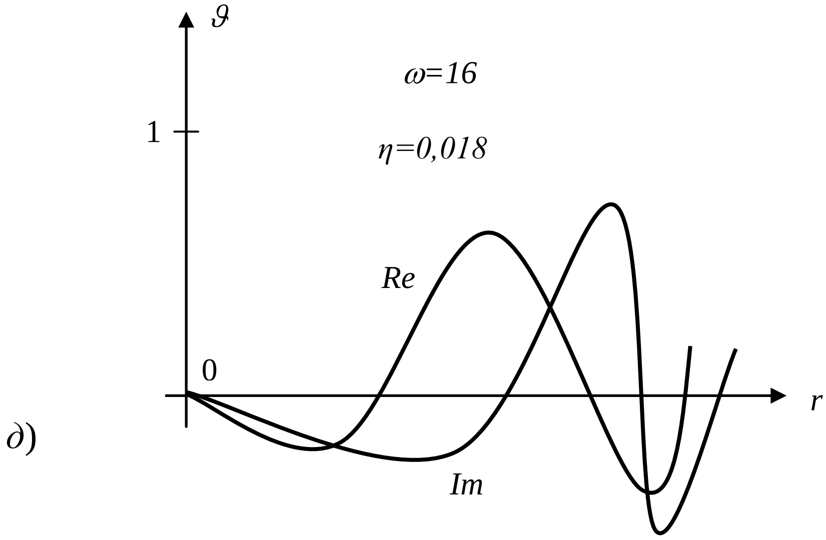

Figure 6. Addiction V the wave number r

В таблице 1 приведены комплексные волновые числа первой моды, полученные методом ортогональной прогонки (колонка 1), путем непосредственного решения уравнения (13) с помощью асимптотических представлений (27) (колонка 2), а также результаты решения уравнения (31) (колонка 3).

Unlike natural oscillations in this case considers the evolution of the two spectral modes with frequency. The first index in the graphs correspond to variations in viscosity, the second index indicates the number mode. For low viscosity rate Rek both modes are close to each other in the low and high frequencies, the phase velocity of the first Su fashion tends to speed in the dry shell. Damping Coefficients grow approximately linearly, and in the second mode, this coefficient is always greater than the first.

In the case of greater frequency of viscosity $Rek_{2,1}$ substantially higher than the frequency $Rek_{2,2}$ on the entire range of variation ω, and the phase velocity $C_γ$ with the growth of ω It tends to infinity. Increasing the viscosity has increased the damping factor for the first mode and the second mode to decrease. In this drawing the first and second damping modes coefficients intersect. A comparison of the waveforms in Fig. 5, the first mode corresponds to the bending vibrations of the shell, and the second primary fluid movements.

Thus, the viscosity facilitates the transfer of motion through the fluid and prevents forward movement through the casing. It is interesting to follow the evolution of their own forms with increasing frequency ω (Second variant of the viscosity). For small values $\omega = \omega_1$ the first shape has an almost linear radially real part and substantially smaller as compared with its imaginary part. The real and imaginary part of the form, comparable in size, have two anchor points (Fig. 5a). Recall that integrated forms of anchor points is not fixed, and some moving areas. With increasing frequency there are nodal points and the first of its own form, one at $\omega = \omega_1$, (Fig. 5b) and three with $\omega = \omega_3$ (Figure 5. c). The number of nodal points of the second form with the growth of does not change the frequency. We note also that the damping coefficient of the second mode is weakly dependent on the frequency.

5. Longitudinally - transverse vibrations

This section analyzes the stationary shell of longitudinal-transverse oscillations of the liquid filled, which in accordance with (5) and (6) can be described as a system of ordinary differential equations of four

$$\begin{aligned} \frac{d\vartheta_r}{dr} &= -\frac{\vartheta_r}{r} - ik\vartheta_z - p \\ \frac{d\vartheta_z}{dr} &= ik\vartheta_r + \frac{1}{\eta\omega}\tau_{\acute{y}} \\ \frac{d\sigma_r}{dr} &= -\rho_0\omega^2\vartheta_z + 2i\eta\omega(\frac{d\vartheta_r}{dr} - \frac{\vartheta_r}{r}) = ik\tau_z \\ \frac{d\tau_z}{dr} &= -\rho_0\omega^2\vartheta_z + 2\eta\omega k(\frac{d\vartheta_r}{dr} - ik\vartheta_z) - ik\sigma_r - \frac{\tau_z}{r} \end{aligned} \qquad (29)$$

With the boundary conditions

$$\begin{aligned} & r = 0: \vartheta_r = 0, \tau_z = 0; \\ & r = R: D\nabla^4 u + \frac{C}{R}(\frac{u}{R} + iv_0 kw) + \sigma_r - \rho_1 h\omega^2 u = 0; \\ & C(iv_0 k\frac{u}{R} - \nabla^2 u) - \tau_z + \rho_1 h\omega^2 w = 0; \; C = \frac{Eh_0}{1-{v_0}^2}. \end{aligned} \qquad (30)$$

P value in the first equation (29) is determined by the basic unknowns according to the expression

$$p = \frac{-\sigma_r + 2i\eta\omega\left(iku + \frac{\vartheta_r}{r}\right)}{\rho_0 C_0^2 - i\omega(k + 2\eta)} \qquad (31)$$

The spectral problem (29), (30), as is the case longitudinal - transverse vibrations were solved by orthogonal shooting. To find the roots of the characteristic equation used Mueller method.

Numerical results. The results of numerical studies of natural oscillations [22]. Figure 7 and Figure 8 shows the dispersion curves $\operatorname{Re}\omega$ and $|\operatorname{Im}\omega|$ the wave number k - the case for incompressible ($C_0 = \infty$ - dot-dash line) and compressible (C_0=0,1 - solid line) of the liquid. Appropriate eigenmodes and k = 0, l and k = 2 are shown in Fig.9. shell and viscosity coefficients of parameters taken following:

h_0 = 0,05; p=1,8; v_0 = 0,25; η =6,011*10(-4); κ=-2 η/3.

Hereinafter are dimensionless quantities for which the units of length and mass density are $R, R\left(\frac{\rho_0}{E}\right)^{\frac{1}{2}}, \frac{1}{\rho_0}$. For an incompressible fluid, there are two modes corresponding to the predominantly longitudinal (curve 1) and predominantly transverse (curve 2) fluctuations in the shell, with complex eigenvalues. All other movements have their own imaginary eigenvalues, that is aperiodic in time. The dashed lines in Figure 7 indicated by the dispersion curves corresponding to the vibrations of a shell with an ideal incompressible fluid. final solution of the problem is given below. Note that unlike the dry shell joint oscillations transverse vibrations to said sheath fluid density ρ_1 It occurs at a lower compared with the longitudinal vibrations frequency in the entire range of wave numbers. When administered viscosity oscillation frequency of the first mode decreases, apparently due to the involvement of additional fluid movement in the masses in the boundary layer, while the second mode appears critical wavenumber restricting bottom region vibrational movements. The paper Vasina, SV, Mikolyuka [11], using the asymptotic methods of solution, the latter effect could not be found. In [17,19] studied the steady oscillations, noted the convergence to zero of the phase velocity of the lowest mode with decreasing frequency. Proper movement envelope and a viscous compressible fluid have an infinite number of modes. Figure 7, Figure 8 are the dispersion curves for the first four modes with minimum frequency oscillations (curves 3,4,5,6). In ascending order of magnitude $\operatorname{Re}\omega$. Comparing curves 1.2 and 3.4 together, you can be sure that a second somewhat worse than the first oscillation mode system shell - compressible fluid to the selected parameters are satisfactorily described by the model of an incompressible fluid in a region of wave numbers $k < 1$. This gives reason to the study of this system in the first approximation we neglect the compressibility of the fluid. elastic shell system - a viscous liquid is dissipativno-inhomogeneous viscoelastic body at the radial coordinate. In this case, unlike the previously discussed torsional vibration is an incompressible fluid, there are two, and compressible - unlimited number of vibrational modes. It is interesting to find out how a synergistic effect may occur in this system. Figure 10, Figure 11 shows the dispersion curves (1.2) for the following parameters shell and liquid:

$$h = 0{,}05; \rho_0 = 80; \nu = 0{,}25t; \eta = 7{,}071 \cdot 10^{-4}; C_0 = \infty$$

Dot-dash lines correspond to fluctuations in the dry shell. The dashed lines show the frequency dependence in the case of an ideal fluid $v^* = 0$. In contrast to the previously discussed options with the density ρ = 8 , in this case, partial frequencies ($v^* = 0$) longitudinal and transverse shell with an ideal fluid oscillation intersect. It is natural to expect that the v^* lying near the intersection point will be a strong coupling of both modes. Dissipative heterogeneous system with similar modes leads to increased power, resulting in a synergistic effect. Indeed, the presence of events shows the effect of the conversion of Vina-longitudinal mode in transverse and longitudinal cross-section in a change of the wave number in the vicinity of the intersection of partial frequencies. As well as a violation of the monotony of growth and synergies. Compared with the previous descriptions of this effect there are two features. Firstly, the effect is far from the place of approximation curves $\operatorname{Re}\omega$ two modes, secondly, damping coefficients of the curves do not intersect. Yu.Novichkov in [12] investigated the coupling joint oscillations of ideal compressible gas and the shell with the aid of diagrams wines. As he examined the frequency of the partial oscillations of gas in rigid walls and an empty shell. Returning to Figure 7, we note a similar manifestation of the effect of guilt in places of convergence 4.5 and 5.6 curves. In these places in Fig.8 synergistic effect is observed for the curves. It is interesting to trace the influence of fluid viscosity on the coupling mode. 3.4 Curves in Figure 10 correspond to the value of the viscosity coefficient η=0,11 at constant other parameters. In this case, primarily transverse mode oscillation is defined on a finite interval of variation of the wave number, and the effect of guilt is not observed, indicating that the loose coupling mode. Another large increase in viscosity (η=0,13, curve 5) leads to the fact that fashion is everywhere transverse vibrations becomes aperiodic, while the longitudinal vibrations appears critical wave number, limiting the scope of the vibrational motions of the top. The physical nature of the observed effect is revealed in the analysis of vibrations of a shell filled with an ideal fluid. The equations of harmonic oscillations of an ideal liquid is easy to deduce from (29), formally putting viscosity coefficients equal to zero.

$$\frac{d\vartheta_r}{dr} = -\frac{\vartheta_r}{r} - ik\vartheta_z - \frac{\sigma}{\rho_0 C_0^2}, \qquad (32)$$
$$\frac{d\sigma}{dr} = -\rho_0\omega^2\vartheta_r,\ \sigma = i\rho_0\omega^2\vartheta_z$$

the general solution of (32) satisfying the condition of unknown limbs at zero, has the form

$$\vartheta_z = AJ_0(qr); \sigma = i\rho_0\frac{\omega^2}{k}AI_0(qr) \qquad (33)$$
$$\vartheta_z = i\frac{q}{R}AI_1(qr); q^2 = \frac{\omega^2}{C_0^2} - k_0^2$$

where A is an arbitrary constant:J_0, J_l,- Bessel functions of zero and first order, respectively. The boundary conditions at r = R are recorded similar to the conditions (29)

$$D\nabla^4 w + \frac{C}{R}\left(\frac{w}{R} + iv_0ku\right) + \sigma_r - \rho_1 h\omega^2 w = 0; \qquad (34)$$
$$C\left(iv_0k\frac{w}{R} - \nabla^2 u\right) + \rho_1 h\omega^2 u = 0;$$

where u - axial movement of the shell, which is now not coincide with the axial movement of the liquid.

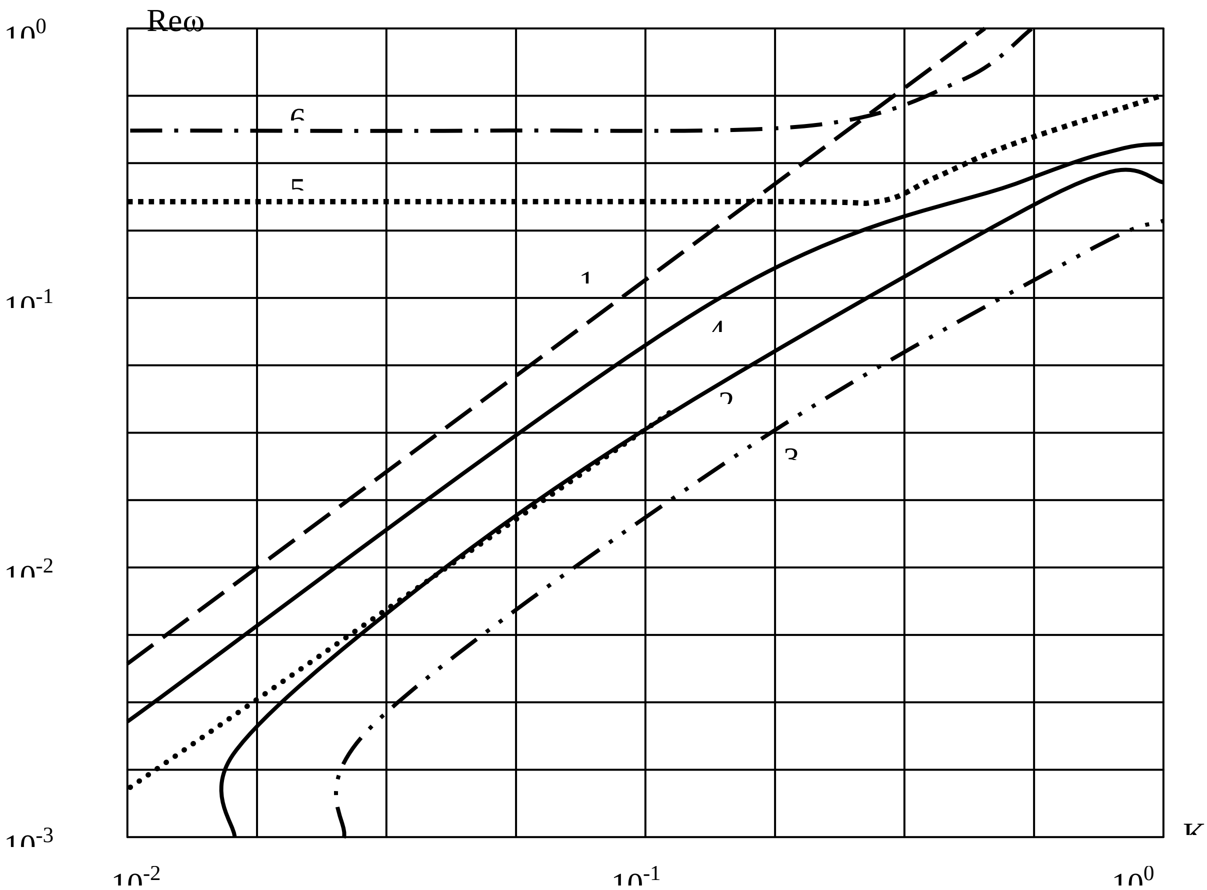

Figure 7. Addiction *Re*ω with a wave of pure K for the case of an incompressible fluid

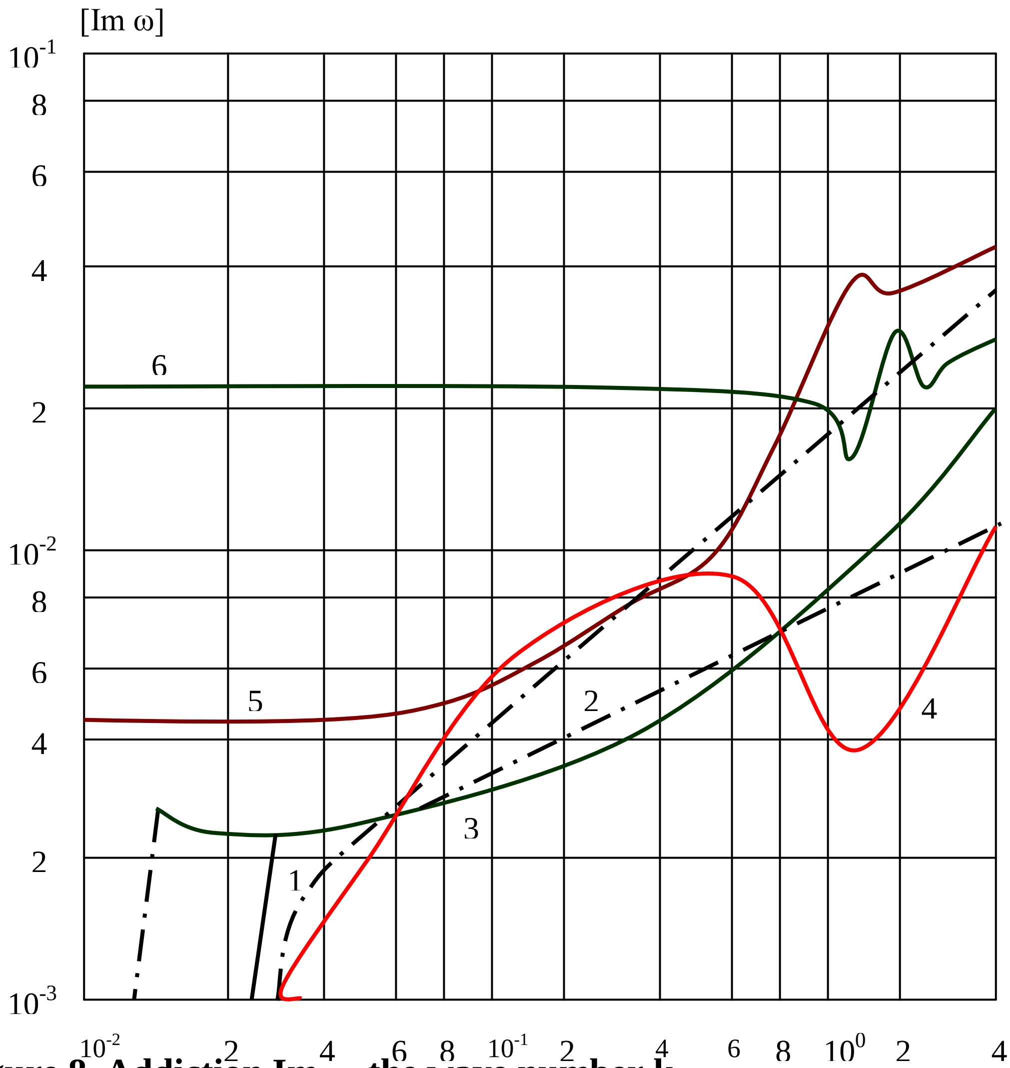

Figure 8. Addiction Im ω the wave number k

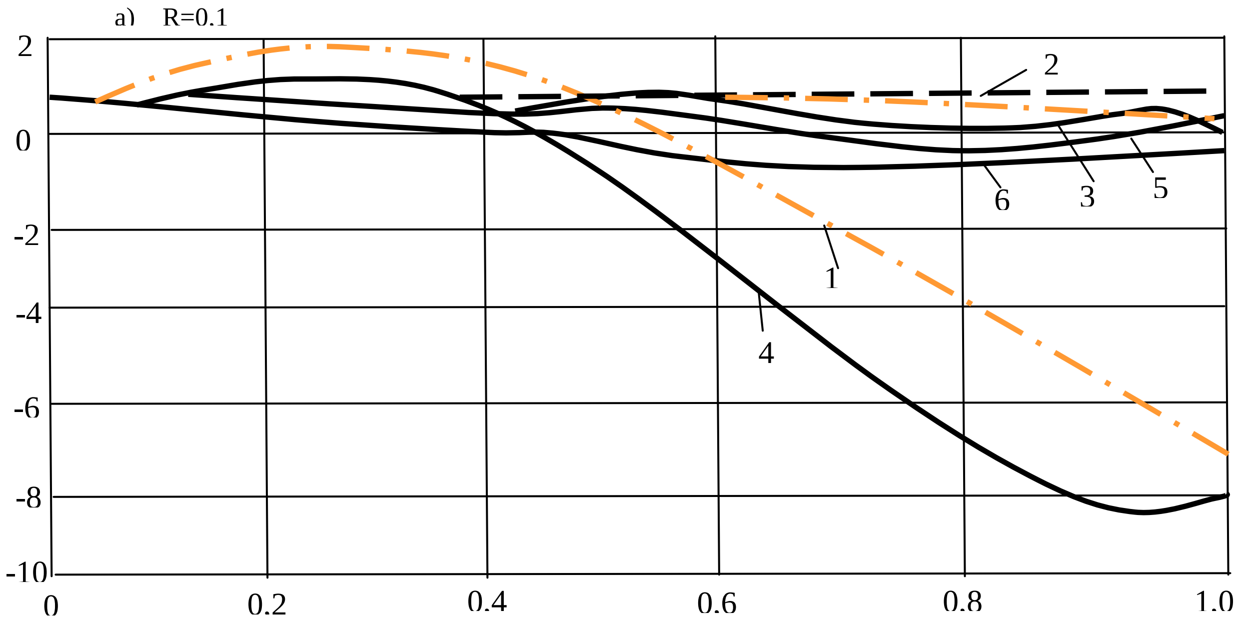

Figure 9 a. Addiction *ReW* the wave number r

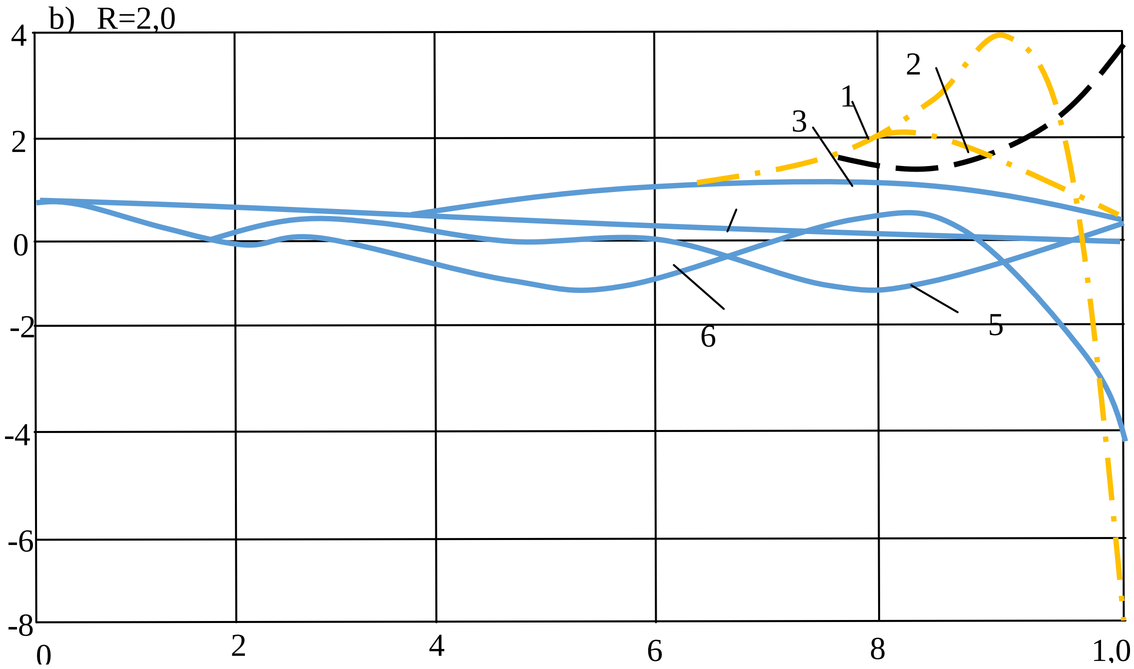

Figure 9, b. Addiction *ReW* the wave number r

After substitution of the solutions (32) of (33) there is a system of linear homogeneous algebraic equations for the unknown A and U_1. The roots of the determinant of this system are the desired eigenvalues, and its decision to determine the relationship between the A and U_1 .

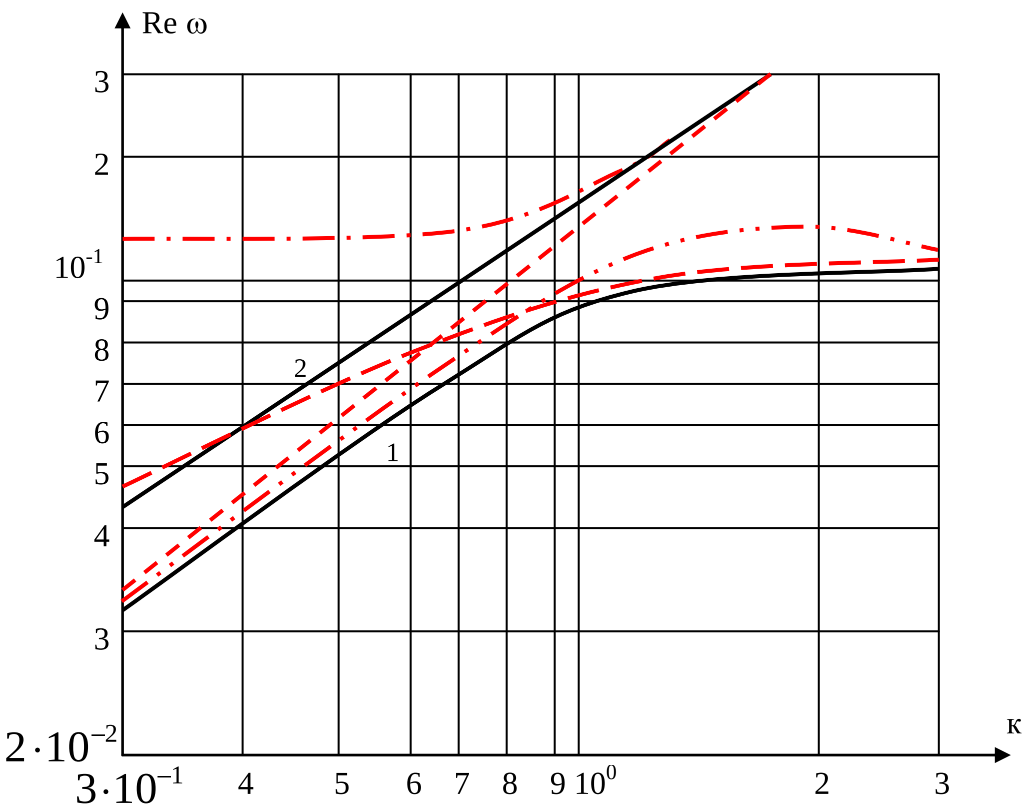

Figure 10. Addiction $Re\omega$ with the wave number k (in the case of an incompressible fluid)

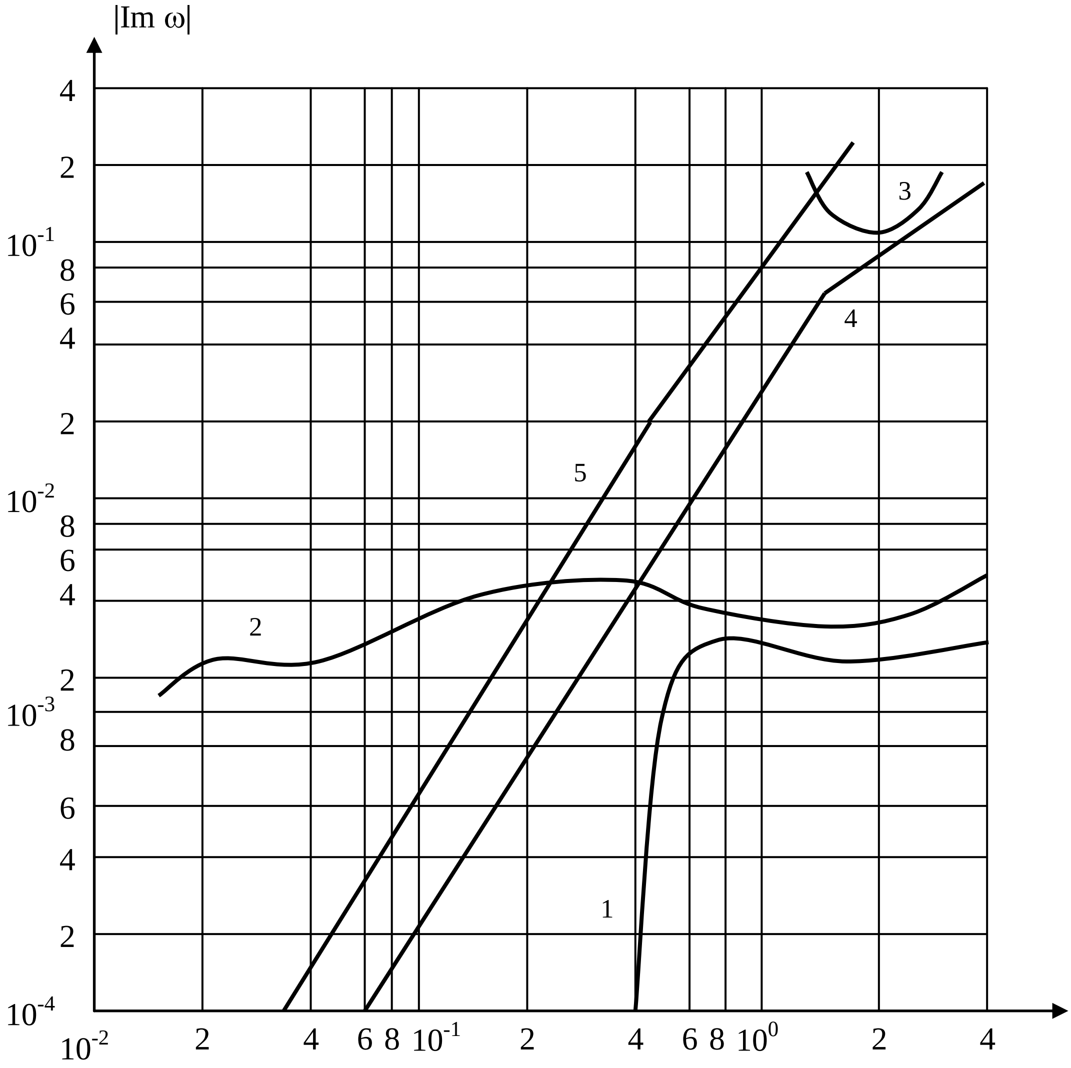

Figure 11. Addiction Im ω the wave number k

For an incompressible fluid, there are two real own Bessel function I_0 and I_1

$$\omega_1 = R\left(\frac{E}{\rho_1}\right)^{\frac{1}{2}}; \omega^2 = \left[\frac{E}{R_1\rho_1}\left(1+h^2R_1\kappa^4\right)/\left(1+\frac{\rho_n I(kR_1)}{h\rho_1 I_1(kR_1)\kappa}\right)\right]^{\frac{1}{2}} \quad (35)$$

In contrast to the dry shell is at the second closing speed is absent, and the phase velocity for small k is equal to size

$$C_R = \left(\frac{Eh}{2\rho_0 R_1}\right)^{\frac{1}{2}} \quad (36)$$

which coincides with the velocity of the Rayleigh wave.

In the case of a compressible fluid $\nu = 0$ and limiting the phase velocity of the mode of transverse vibrations of a shell at k —> 0 is the speed of waves Korteweg - Zhukovsky.

$$C_k = \frac{C_0 C_R}{(C_0^2 + C_R^2)^{\frac{1}{2}}} \qquad (37)$$

Numerical study showed that the critical value C_k It is independent of fluid viscosity, but with increasing η Poisson's ratio is weakened dependence oscillations, so that the ratio $(\max im\varpi)/(\min im\varpi) \to 1$ and a private U-shape becomes flat. As follows from the results, in general, within the framework of an engineering statement of the problem, can not adequately describe the longitudinal vibrations of a cylindrical shell filled with a viscous fluid with the help of the core theory.

Conclusions

1. Numerical study showed that the critical value V_k It is independent of fluid viscosity, but with increasing η Poisson's ratio is weakened dependence oscillations, so that the ratio $(\max im\omega)/(\min im\omega) \to 1$ and a private U-shape becomes flat. However, in some special cases, namely, when high viscosity or a critical value of the Poisson ratio can provide a method of natural frequency estimate based on the core model of the type (26).
2. Analyzing the dependence of the energy dissipation of the wave number, it should be noted that there are two opposing trends, with the growth of of the wave number at a fixed amplitude *v*, linearly increasing shear stresses $p_{z\varphi}$. And also, as evidenced by the numerical results of simultaneous localization fluid movement near the amplitude envelope resulting in a reduction in the mass of liquid involved in the movement and shear stresses $p_{r\varphi}$.
3. For low-frequency viscosity Re*k* Both modes are close to each other in the low-frequency range, and the phase velocity at high frequencies C_y the first mode tends to speed in the dry shell. Damping Coefficients grow approximately linearly, and in the second mode, this coefficient is always greater than the first. In the case of greater frequency of viscosity Re$k_{2,1}$ substantially higher than the frequency Re$k_{2,2}$ on the entire range of variation ω , and the phase velocity C_y with a growth tends to infinity.

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